# Principal Component Analysis of Cepheid Variable Stars

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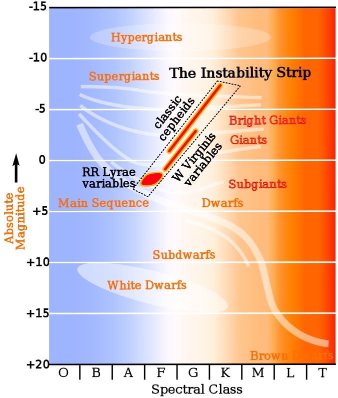
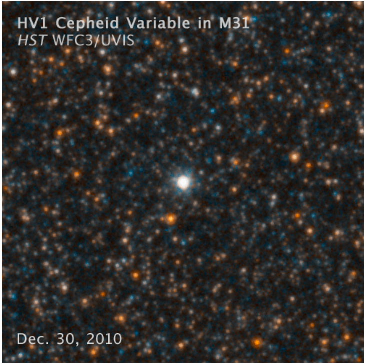
# Variable Stars



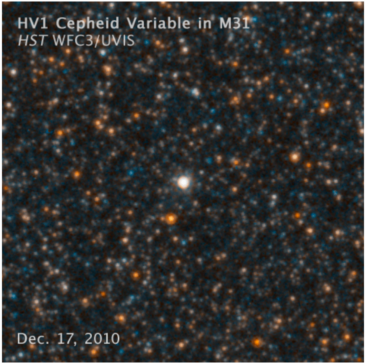

- stars whose size and luminosity are variable
- happens almost exclusively in giants
- occurs mainly in stars which lie in the instability strip

# Classical Cepheid Variable Stars




- population I (high metallicity)
- 4–20 solar masses
- up to 100000 solar luminosities
- pulsation period can range from days to months
- period-luminosity relationship

# Classical Cepheid Variable Stars

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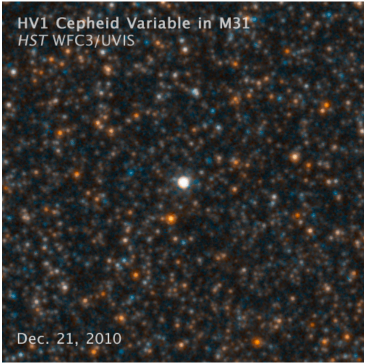


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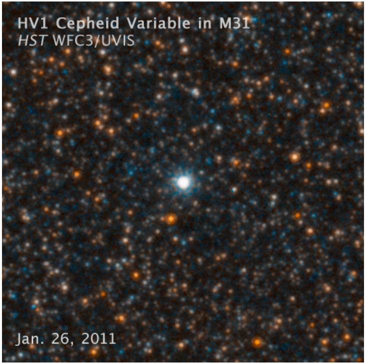


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# Lightcurves

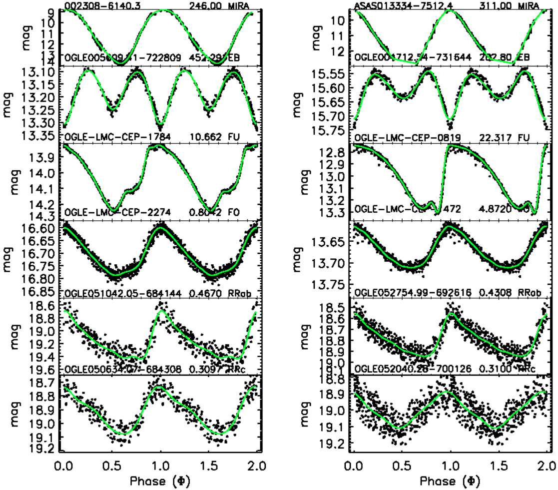
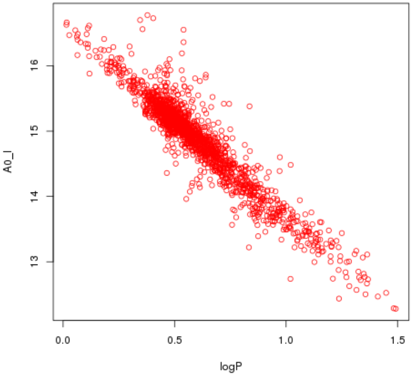



Figure: Lightcurves of different classes of variable stars

# Cepheid Period-Luminosity Relationship



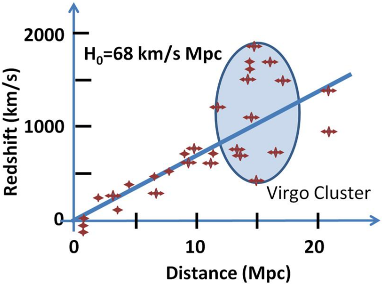

- a Cepheid's period of oscillation is related to its mean luminosity
- approximate a linear model which gives $a$ and $c$
- this makes $A_0$ a function of $\log P$ and some constants

$$\underbrace{m}_{\text{apparent magnitude}} - \underbrace{M}_{\text{absolute magnitude}} = 5 \log \left( \frac{d}{10} \right) - 5$$

$$\underbrace{d}_{\text{distance in parsecs}} = 10^{\frac{m-M}{5}+1}$$

$$\underbrace{A_0}_{\text{mean magnitude}} = a \log \underbrace{P}_{\text{period}} + c$$

# Hubble's Law




$$\underbrace{v}_{\text{redshift velocity}} = \underbrace{H_0}_{\text{Hubble's constant}} \underbrace{d}_{\text{distance}}$$

- Hubble's law describes the velocity of the expansion of the Universe
- redshift measurements give us $v$
- Cosmic Microwave Background (CMB) only gives us a measure of $H_0^2 \Omega$
- independent measure of $H_0$ is needed to find density of Universe, $\Omega$

# Fourier Analysis of Lightcurves

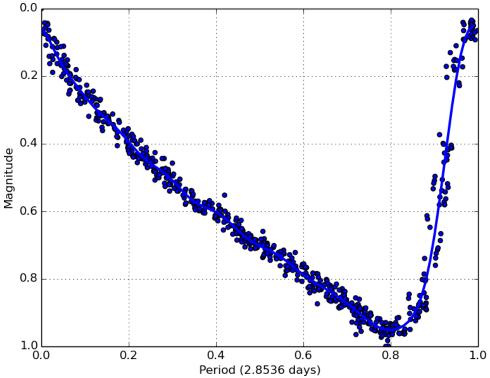



Figure: Fundamental Mode Cepheid in the LMC with 7th order Fourier fit from OGLEIII

- assume basis lightcurve to be sinusoidal
- find values of best fit for $A_0$, $A_k$ and $\Phi_k$
- for $n^{\text{th}}$order fit, requires $2n + 1$ parameters

$$\underbrace{A(t)}_{\text{mag. at time } t} = \underbrace{A_0}_{\text{mean mag}} + \sum_{k=1}^{n} \underbrace{A_k}_{\text{scaling } \updownarrow} \sin(\underbrace{k}_{\text{scaling } \leftrightarrow} \omega t + \underbrace{\Phi_k}_{\text{shift } \leftrightarrow})$$

# Fourier Parameters versus $\log P$

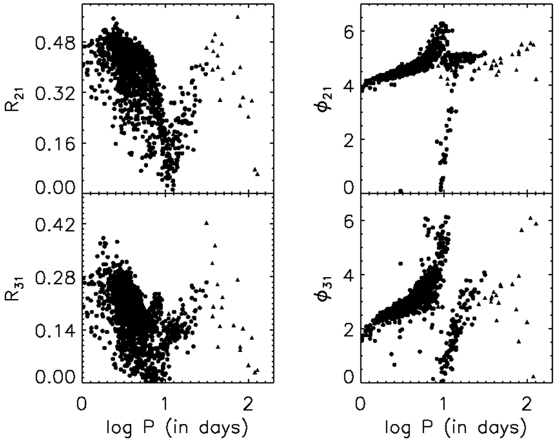



Figure: Fourier parameter ratios of 1829 fundamental mode Cepheids in LMC

# Principal Component Analysis of Lightcurves

- data decides the basis lightcurves
- construct a matrix of all the stars' lightcurves stacked vertically
- find the covariance matrix of this matrix $(\mathbf{A}^T\mathbf{A})$
- eigenvectors $(\mathbf{EV})$ of the covariance matrix are the basis lightcurves
- scalar coefficents are the principle scores $(PC)$
- $n^{\text{th}}$order fit requries only $n$ parameters for each star, in addition to the $n$ eigenvectors for the whole dataset

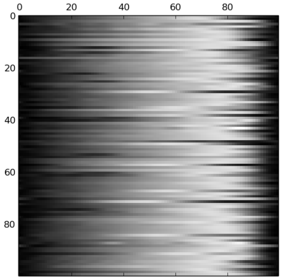



Figure: First 100 rows of input matrix

$$PC_i = \mathbf{A} \cdot \mathbf{EV}_i$$

$$\mathbf{A} = \sum_{i=1}^{n} PC_i \mathbf{EV}_i$$

# Principal Component Analysis of Lightcurves

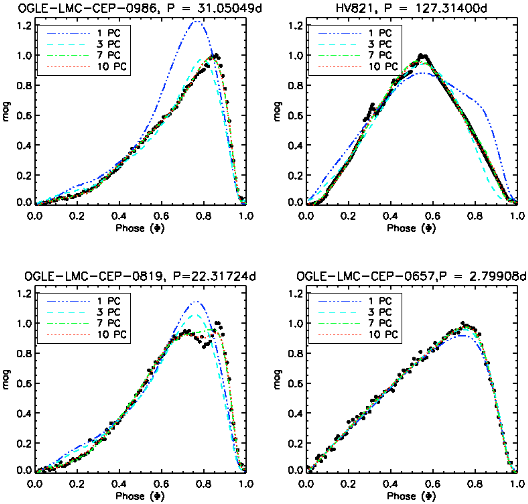



Figure: Cepheids with varying order PCA fits

# Principal Scores versus $\log P$

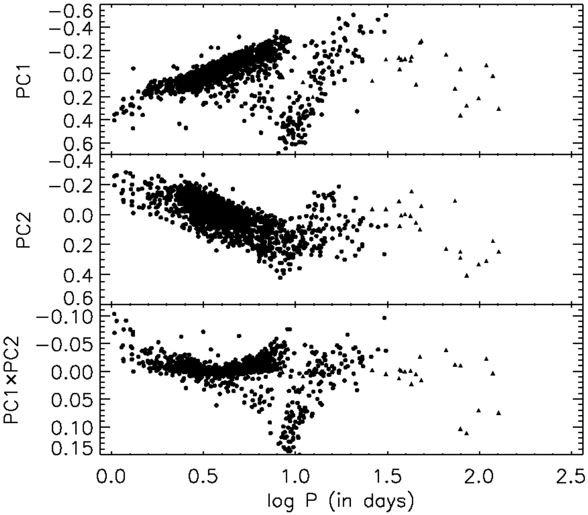
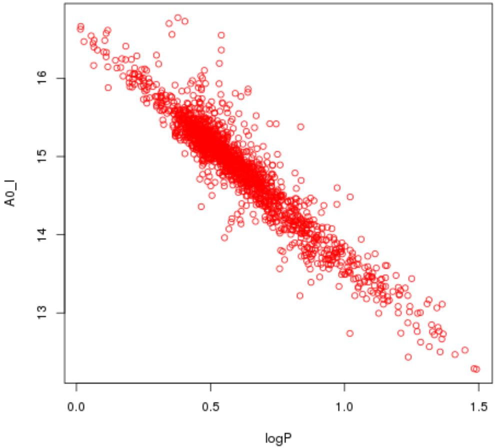


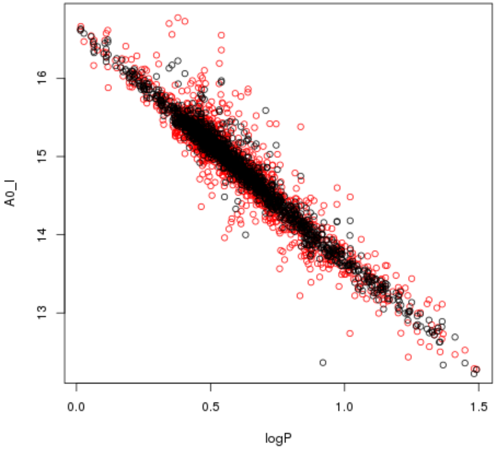

Figure: Principal scores 1 and 2 as functions of $\log P$ for 1829 fundamental mode Cepheids in LMC

- $A_0 = a \log P + c$



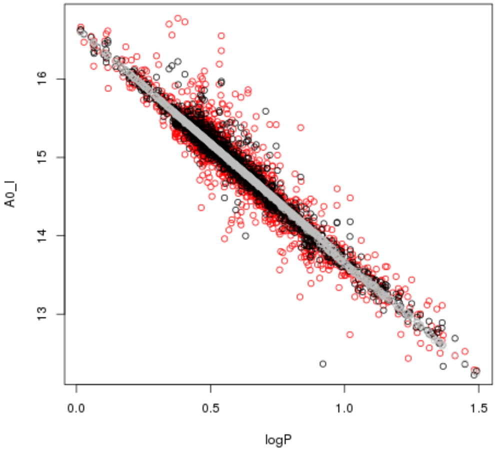
# Cepheid Period Luminosity Color Relationship

- $A_0 = a \log P + c$
- $A_0 = a \log P + b(B - V) + c$



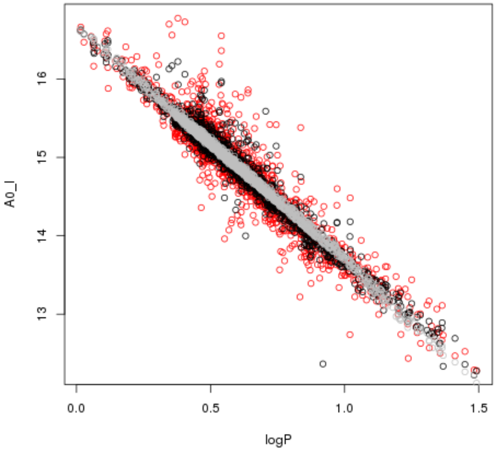
# Cepheid Period Luminosity Principal Component Relationship

- $A_0 = a \log P + c$
- $A_0 = a \log P + b(B - V) + c$
- $A_0 = a \log P + bPC_1 + c$



# Cepheid Period Luminosity Principal Component Relationship

- $A_0 = a \log P + c$
- $A_0 = a \log P + b(B - V) + c$
- $A_0 = a \log P + bPC_2 + c$



# Period Luminosity Principal Component Relationship

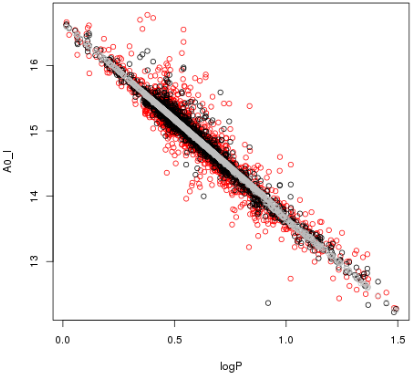



Figure: $A_0$ fitted with $PC_1$ vs $\log P$

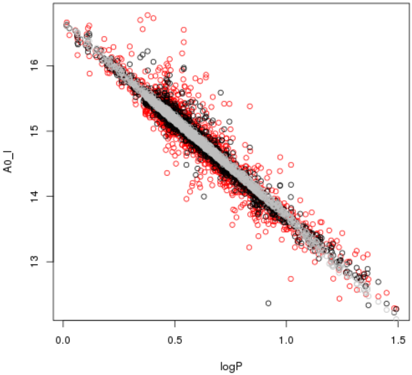



Figure: $A_0$ fitted with $PC_2$ vs $\log P$

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