The Precession of Mercury’s Perihelion
Where Newton failed and Einstein succeeded.

Dan Wysocki
SUNY Oswego

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many models were accurate during the lifespan of the people who created them, but over long periods of time began to lose accuracy
eventually a new model comes along which takes its place, until yet another model replaces that one
Geocentric Model of the Universe

- most ancient view of the Universe
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- Earth is located at the center of the Universe
Most ancient view of the Universe

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- All stars and planets move about the Earth
stars lie on a great sphere which encircles the Earth, and rotates once per day
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these planets were thought to circle the Earth in their own paths, presumably closer to Earth than the sphere
the planets’ orbital paths appear to make small loops in their otherwise circular orbits
Retrograde Motion of the Planets

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- difficult to explain this in a geocentric model
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- most explanations involved “epicycles”
Epicycles

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- slowly they would go out of sync with observations
Geocentric Complexity
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Mercury Precession
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Proper Motion

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- Apart from that, the stars had all drifted in seemingly random directions and speeds.
- Some of the brighter stars had drifted by almost an entire degree.
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not a perfect model, but was able to simplify the motions of the planets
Copernican Retrograde Motion

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  - both cars are moving forward, but the slower car appears to be moving backwards
Kepler’s Laws

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Kepler’s Laws

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- corrects Copernicus’ false assumption that orbits are circular, when they are in fact elliptical
Kepler’s First Law

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- $r$ – a planet’s distance from the Sun

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$$r = r_0 \frac{1 + \varepsilon}{1 + \varepsilon \cos \phi}$$ (1)
Kepler’s Second and Third Laws

- **Kepler’s second law**
  - orbit sweeps out equal areas in equal times

\[ \frac{dA}{dt} = \text{constant} \]

\[ P^2 = \frac{4\pi^2}{\text{constant}} a^3 \]
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Newtonian Gravity

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- The constant in Kepler’s second law was solved for.

Mathematically, this is represented by the equations:

\[ \frac{dA}{dt} = \frac{h}{2} \]

and

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Orbital Precession

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- Newton’s law of gravity predicts this, while Kepler’s laws do not.
Newtonian Precession Predictions

Newton’s laws do a good job approximating the rate of precession for all the planets except Mercury.

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**Table:** Precession rates in arc-seconds per century
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- Error for the other planets can be explained by imprecision of measurements, but Mercury’s is too large.
- Mystery puzzled physicists for many years.
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- a new explanation was in order
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- rejected the notion that gravity was a force
- spacetime itself is bent in the presence of mass
- the planets aren’t travelling in a curved path around the Sun, the space around the Sun is itself curved
using general relativity, we can adapt Newton's law of gravity to be more precise
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this was one of the first things that Einstein calculated to test his theory out
Conclusions

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- sometimes a completely new approach is needed to be successful
URL: http://www.astronomy.ohio-state.edu/~pogge/Ast162/Movies/uma.gif.
URL: https://lh3.ggpht.com/-cgMuSQK8oEg/TqnJBliUhXI/AAAAAAAAANk/yRe1t5WUbzo/s1600/old+geocentric+model.jpg.
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Halley, Edmund. “Considerations on the Change of the Latitudes of Some of the Principal Fixt Stars”. In: *Philosophical Transactions of the Royal Society* 30 (1717).


Questions?