

The Precession of Mercury's Perihelion

Where Newton failed and Einstein succeeded.

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QUEST 2014

Introduction

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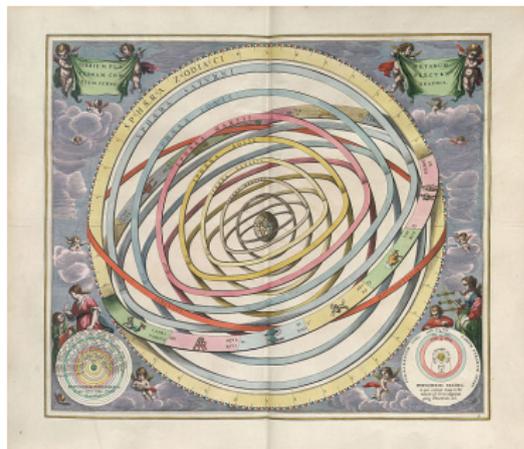
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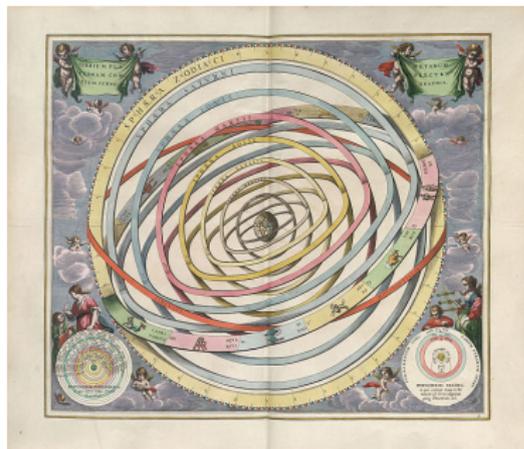
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- many models were accurate during the lifespan of the people who created them, but over long periods of time began to lose accuracy
- eventually a new model comes along which takes its place, until yet another model replaces that one

Geocentric Model of the Universe



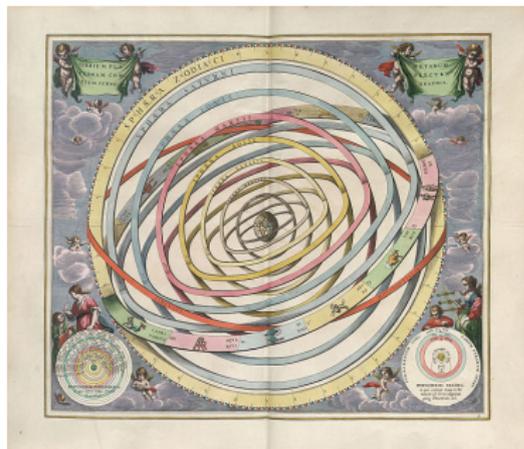
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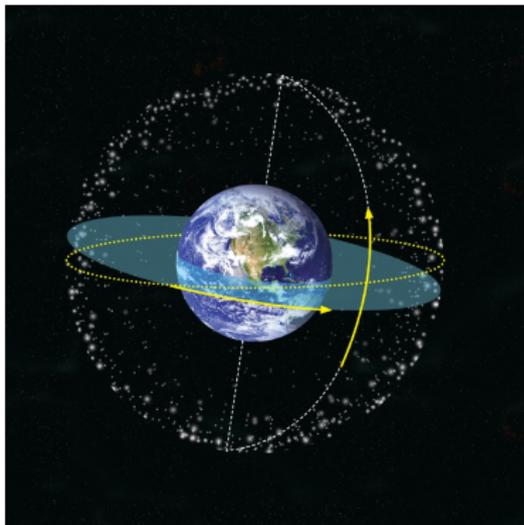
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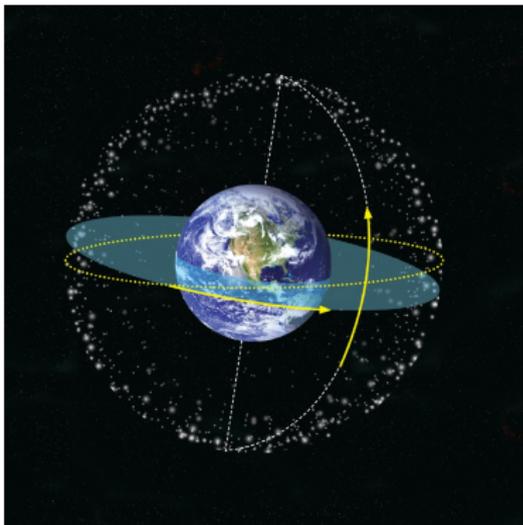
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- all stars and planets move about the Earth

Celestial Sphere and Planets



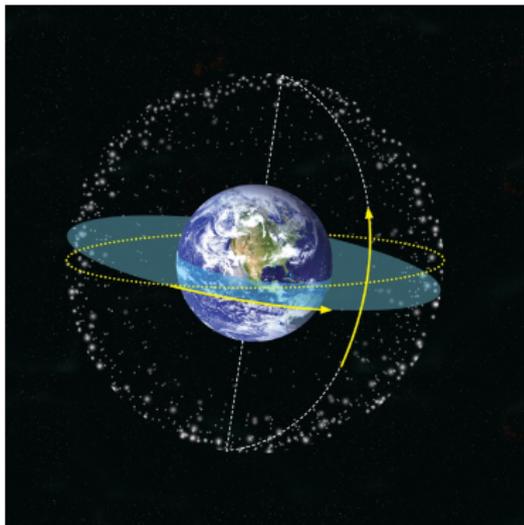
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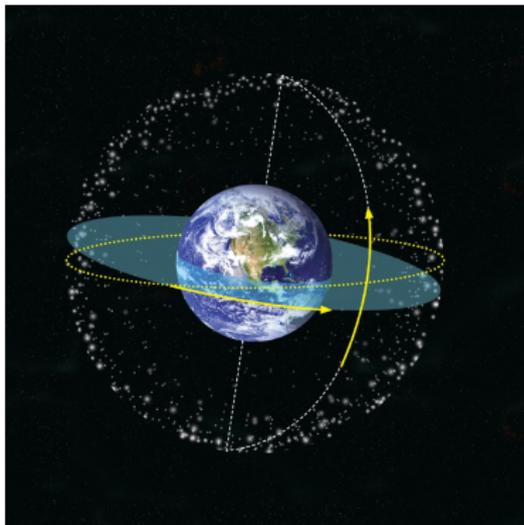
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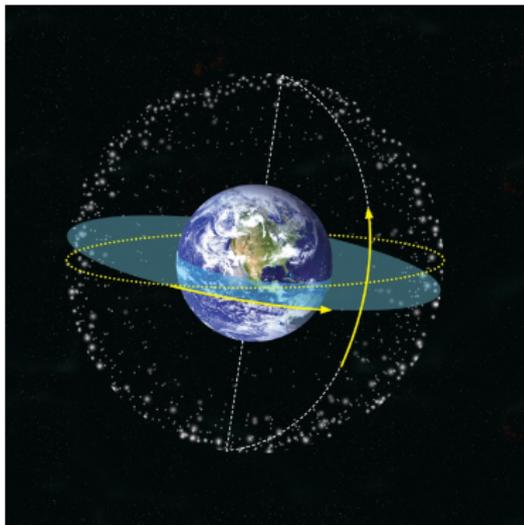
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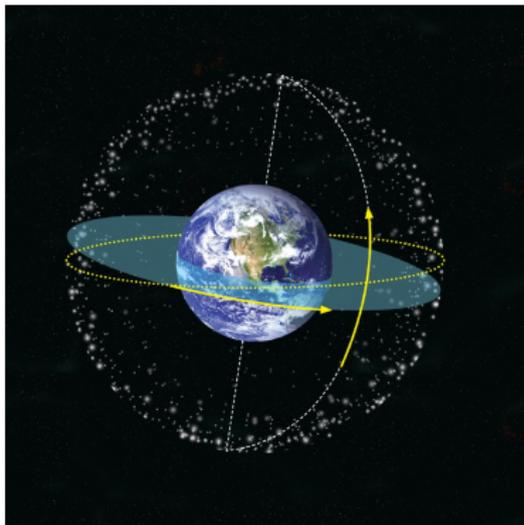
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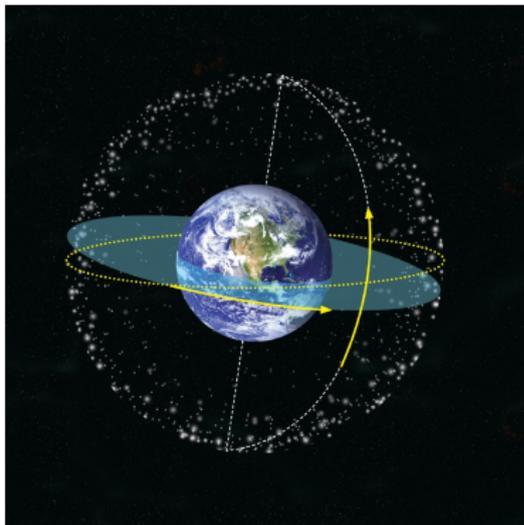
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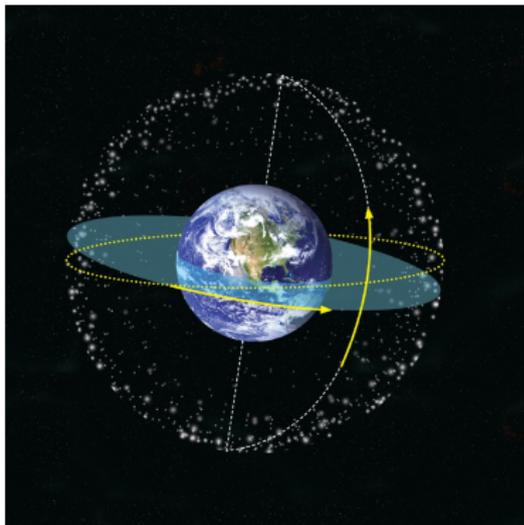
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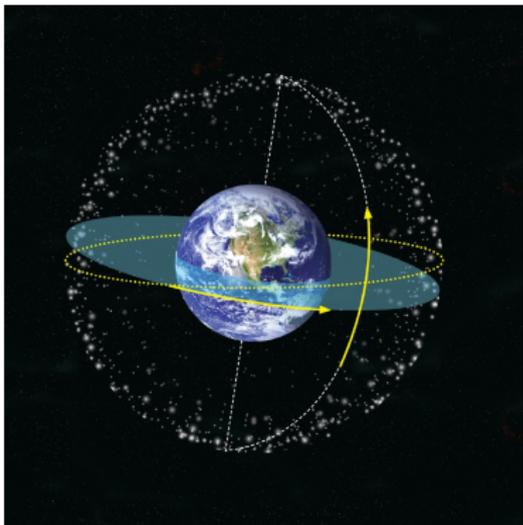
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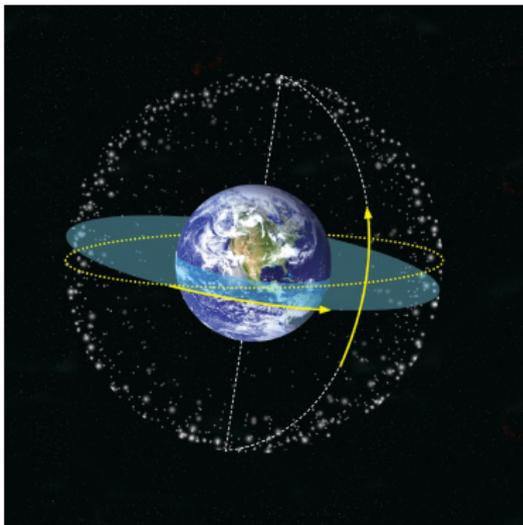
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 - the Sun, the Moon, Mercury, Venus, Mars, Jupiter, and Saturn
- these planets were thought to circle the Earth in their own paths, presumably closer to Earth than the sphere

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- most explanations involved “epicycles”

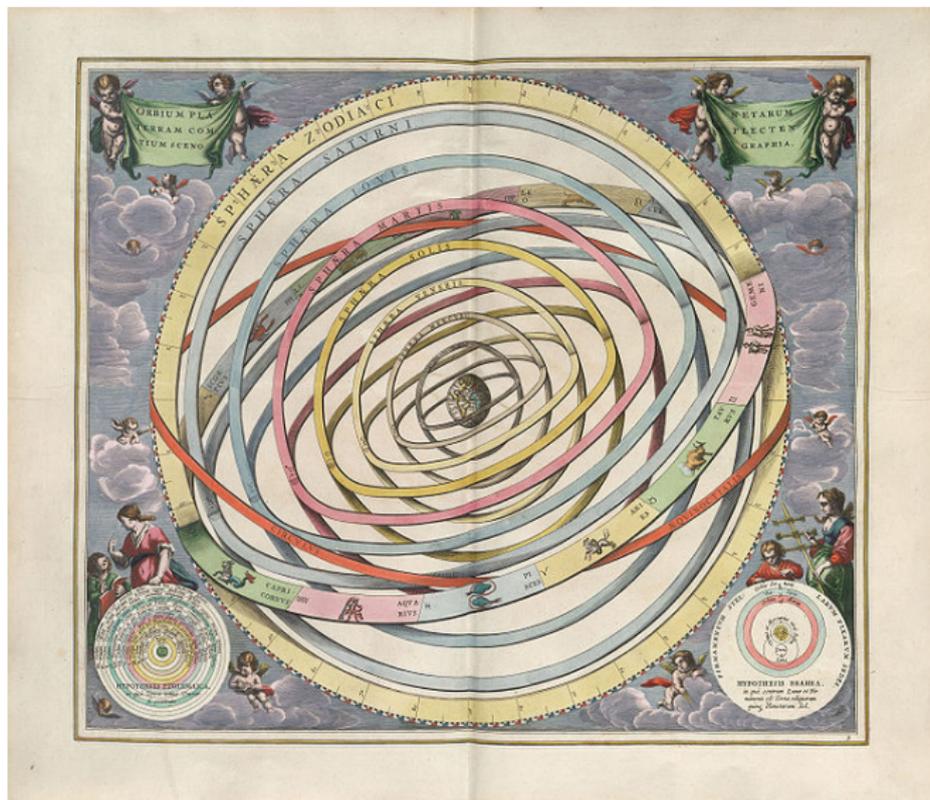
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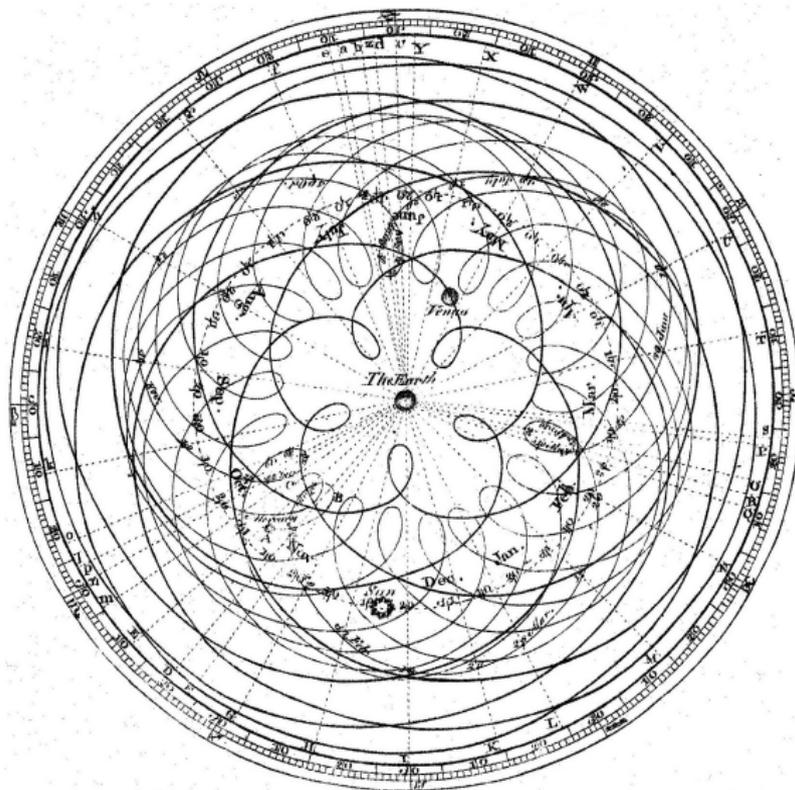
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- slowly they would go out of sync with observations

Geocentric Complexity



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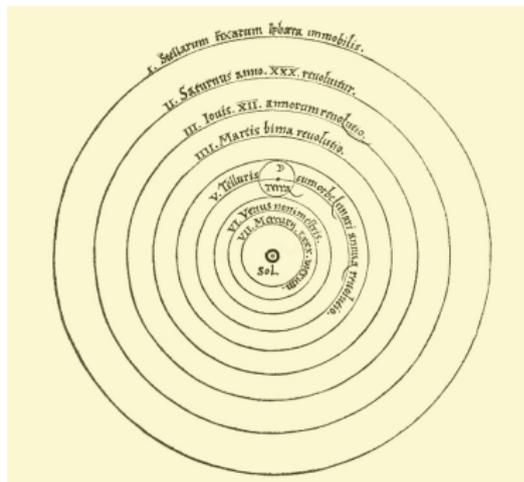
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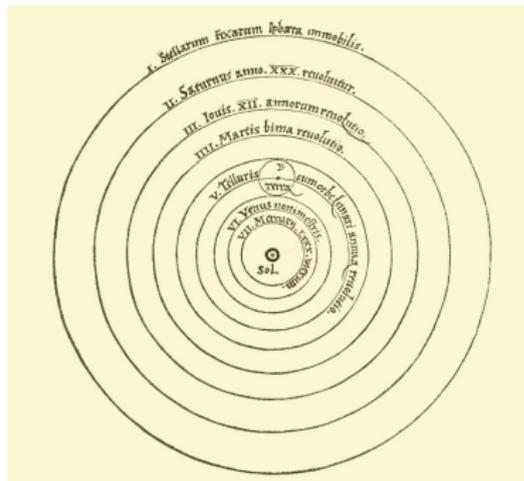
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- some of the brighter stars had drifted by almost an entire degree

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- not a perfect model, but was able to simplify the motions of the planets

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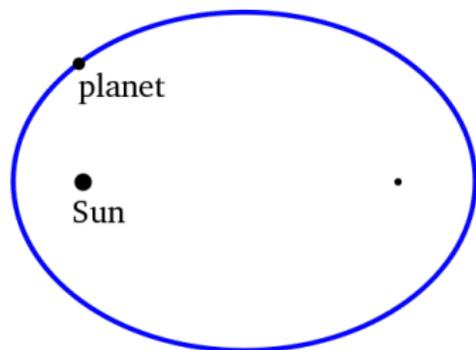
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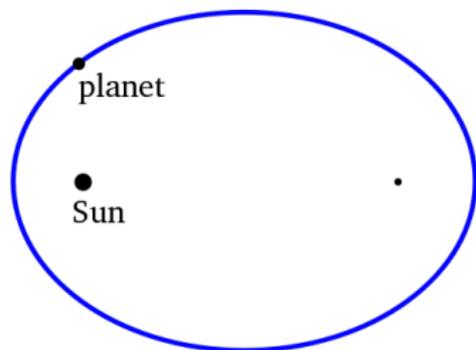
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 - both cars are moving forward, but the slower car appears to be moving backwards

Kepler's Laws



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- corrects Copernicus' false assumption that orbits are circular, when they are in fact elliptical

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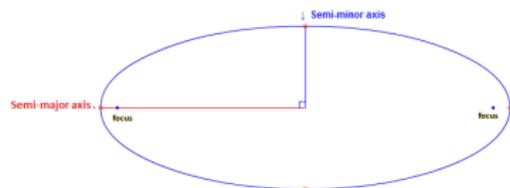
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$$r = r_0 \frac{1 + \varepsilon}{1 + \varepsilon \cos \phi} \quad (1)$$

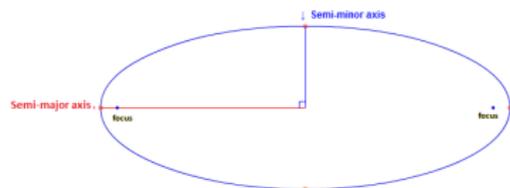
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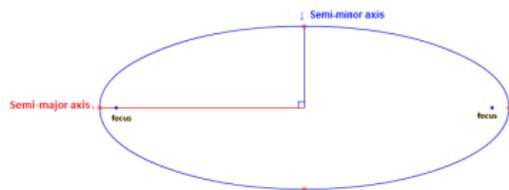
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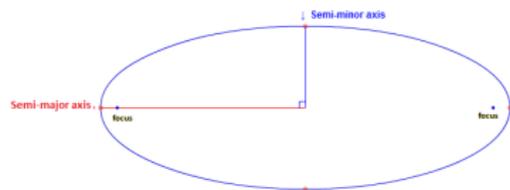


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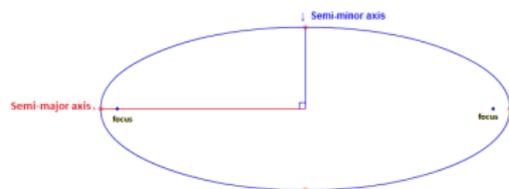


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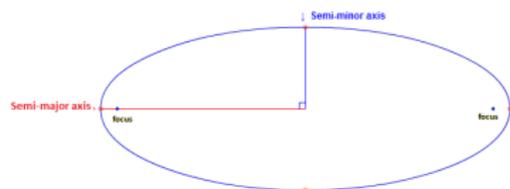
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- Newton's law of gravity predicts this, while Kepler's laws do not

Newtonian Precession Predictions

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Mercury	575.19''	532.08''
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Table: Precession rates in arc-seconds per century

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- mystery puzzled physicists for many years

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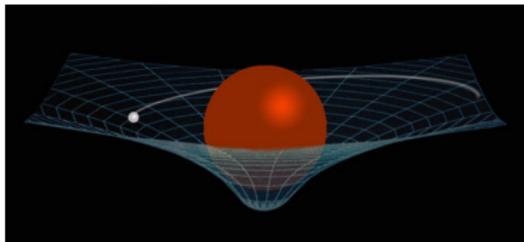
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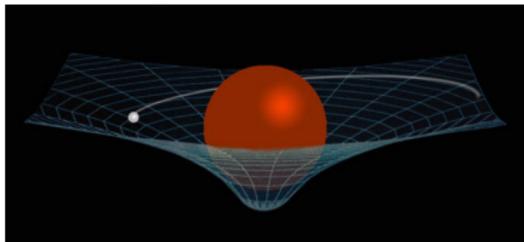
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- a new explanation was in order



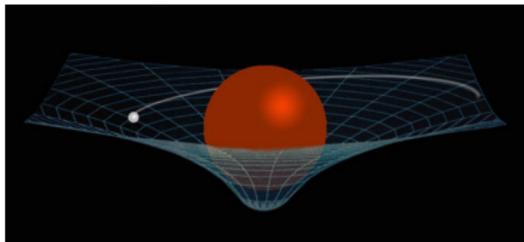
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General Relativity



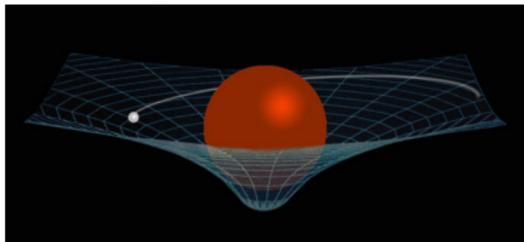
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- the planets aren't travelling in a curved path around the Sun, the space around the Sun is itself curved

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- this was one of the first things that Einstein calculated to test his theory out

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- sometimes a completely new approach is needed to be successful

References I

URL: <http://www.spaceanswers.com/wp-content/uploads/2013/01/Celestial-sphere-illus.jpg>.

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Questions?